Localization in Dynamical Mean Field Theory
Lecture 2: Friedel Oscillations and Electronic Griffiths Phases

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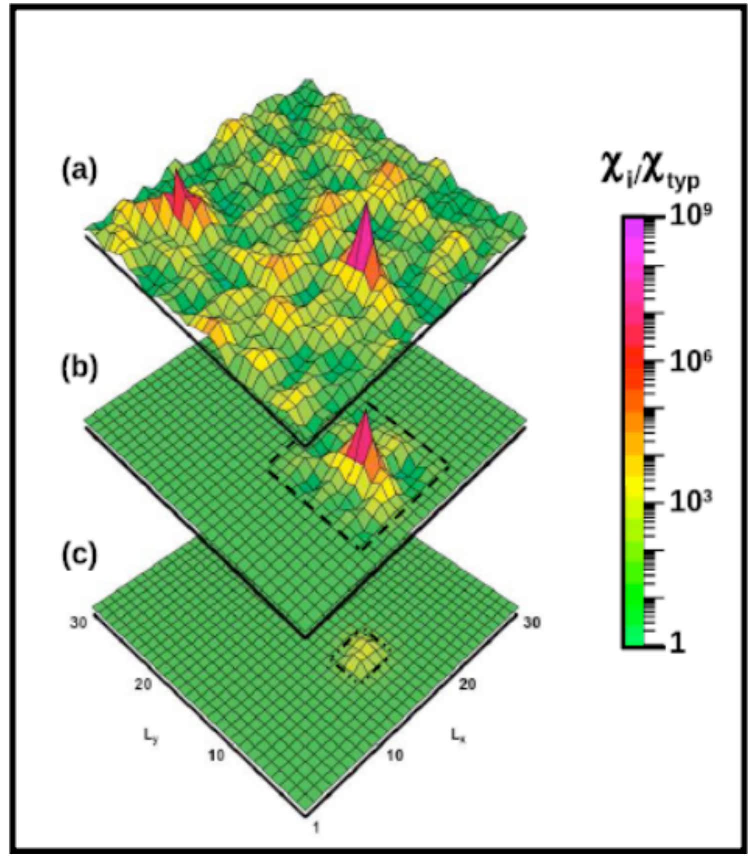


NATIONAL HIGH MAGNETIC FIELD LAB

Workshop "Localization in Quantum Systems"
Jun. 1-2, 2017, King's College London

Local perspective: the cavity field?

(Abou-Chacra, Thouless, Anderson (1973))

Local effective action (expanded to O(t^2)): **Anderson impurity model**

$$S_{\mathrm{eff}}(i) = S_{\mathrm{loc}}(i) - \ln \Xi(i)$$

$$= \sum_{\sigma} \int_{o}^{\beta} d\tau \int_{o}^{\beta} d\tau' \, c_{i,\sigma}^{\dagger}(\tau)$$

$$\times [\delta(\tau - \tau')(\partial_{\tau} + \varepsilon_i - \mu) + \boxed{\Delta_{i,\sigma}(\tau,\tau')]}$$

$$\times c_{i,\sigma}(\tau') + U \int_{o}^{\beta} d\tau \, n_{i,\uparrow}(\tau) n_{i,\downarrow}(\tau).$$

"cavity field"?

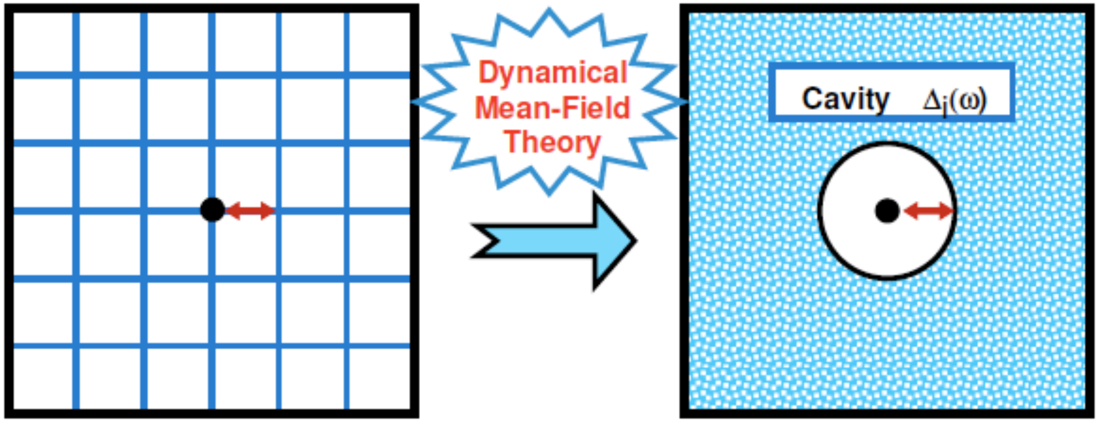
$$\Delta_i(\omega_n) = \sum_{j=1}^{z} t_{ij}^2 G_j^{(i)}(\omega_n)$$

fluctuates
in energy and space

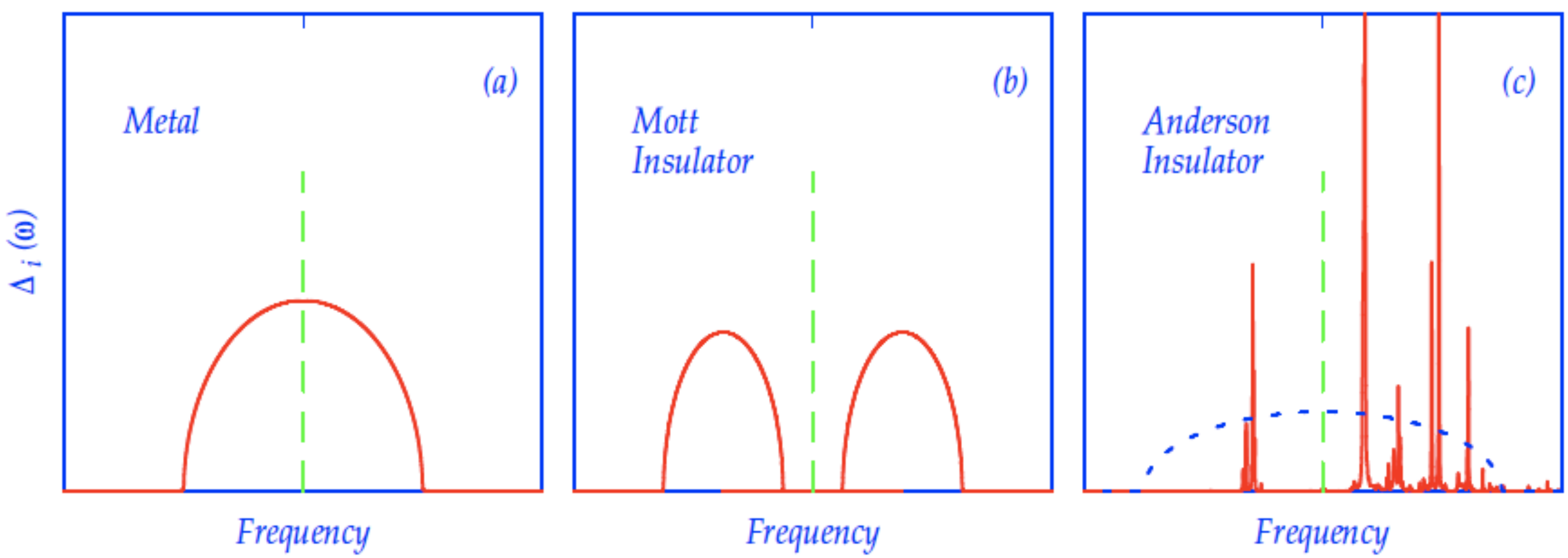
Recursion relation:

$$G_{cj}^{(i)(-1)}(\omega) = \omega - \varepsilon_j - \sum_{k=1}^{z-1} t_{jk}^2 G_{ck}^{(j)}(\omega)$$

Fluctuating cavity field

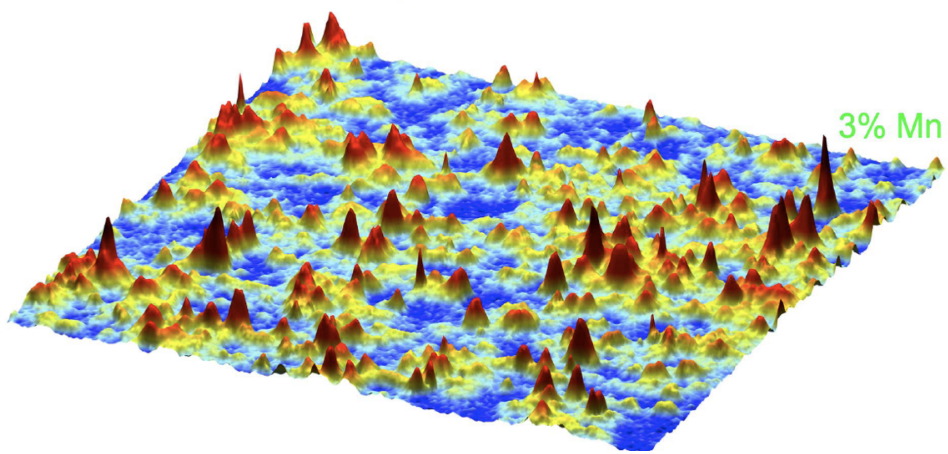


Bethe lattice simulation



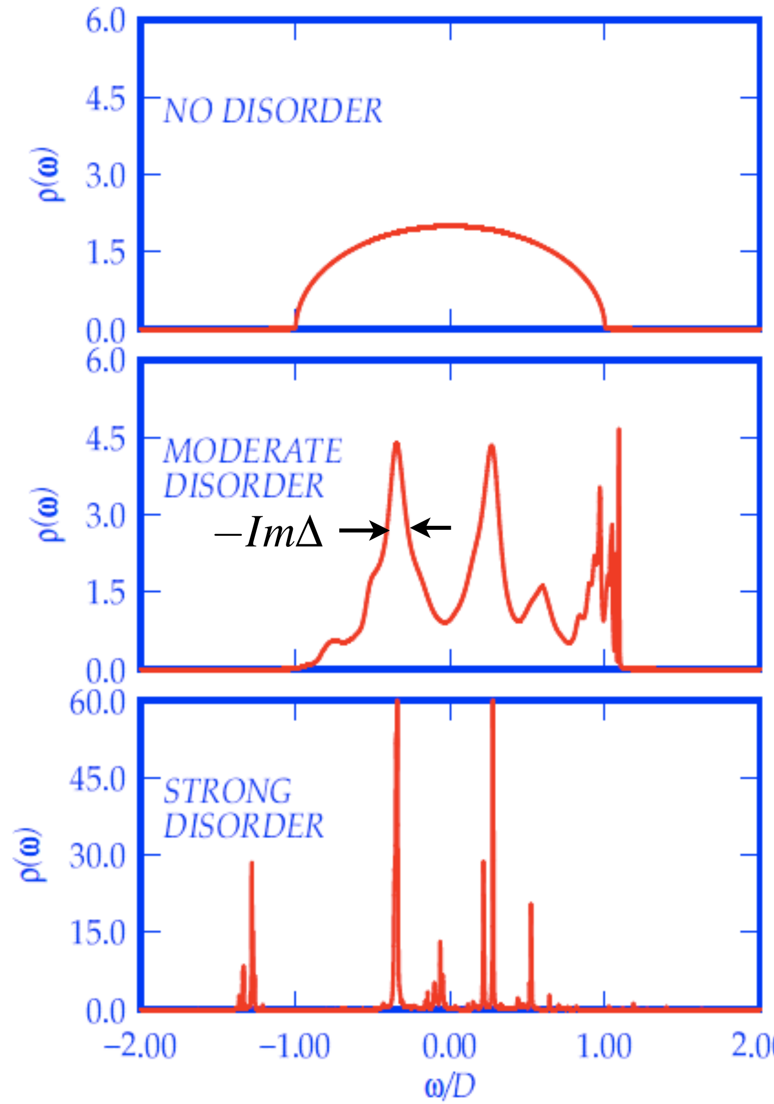
Can local spectrum recognize Anderson localization?

$$\rho_i(\omega) = \frac{1}{\pi} Im \frac{1}{\omega - \varepsilon_i - \Delta_i(\omega)}$$
$$= \sum_n \delta(\omega - \omega_n) |\psi_n(i)|^2$$



Yazdani, STM experiments GaMnAs
(close to localization)

Bethe lattice simulation

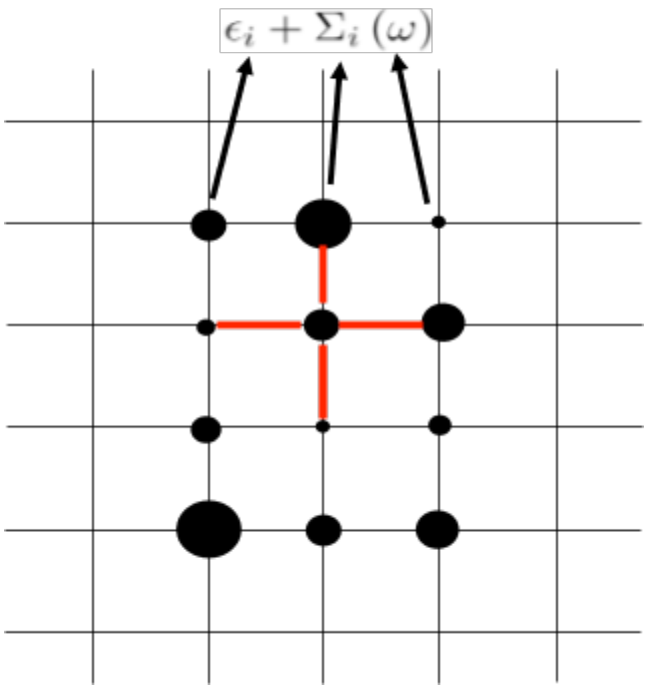


Disordered Mott Transitions: Quantum TAP

• Clean case (W=0): Mott metal-insulator transition at U=U_c, where $\boldsymbol{Z} \rightarrow 0$ (Brinkman and Rice, 1970).

• Fermi liquid approach in which each fermion acquires a quasi-particle renormalization and each site-energy is renormalized:

Local renormalizations



$$\Sigma_i(\omega) = \left(1 - Z_i^{-1}\right)\omega - \varepsilon_i + \bar{\varepsilon}_i / Z_i$$

Local moment formation: $Z_i \longrightarrow 0$

Orbitally (site) selective Mott transition?

"deconfinement", "fractionalization"

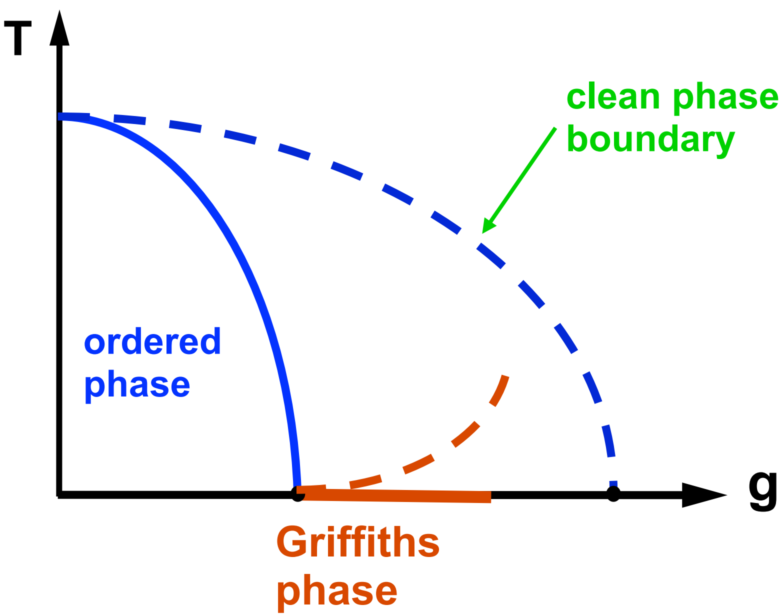
"Kondo" THEOREM: in any metal

$Z_i \neq 0$ $\rho_i \neq 0$ **(continuum spectrum)**

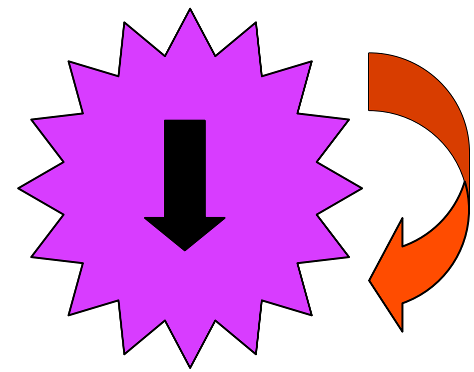
(exceptions on Friday)

Quantum Griffiths phases and IRFP (1990s)

• **D. Fisher (1992): new scenario for (insulating) QCPs with disorder (Ising)**



Griffiths phase (Till + Huse):



Rare, dilute magnetically ordered cluster tunnels with rate $\Delta(L) \sim \exp\{-AL^d\}$

$P(L) \sim \exp\{-\rho L^d\}$

$P(\Delta) \sim \Delta^{\alpha-1}$; $\chi \sim T^{\alpha-1}$

$\alpha \to 0$ at QCP (IRFP)

E.Miranda, V. Dobrosavljevic,
Reports on Progress in Physics 68, 2337 (2005)

stat-DMFT: results in D=2

PRL 102, 206403 (2009) PHYSICAL REVIEW LETTERS week ending 22 MAY 2009

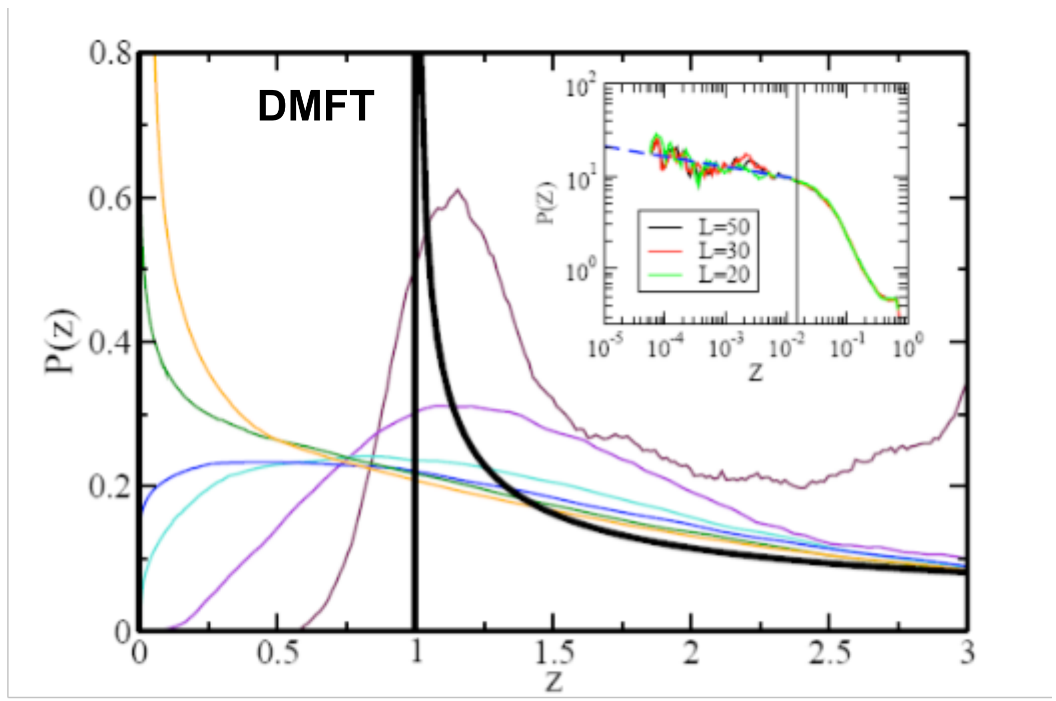
Electronic Griffiths Phase of the $d = 2$ Mott Transition

E. C. Andrade,[1,2] E. Miranda,[2] and V. Dobrosavljević[1]

- In D=2, the environment of each site (“bath”) has strong **spatial fluctuations**
- **New physics**: **rare evens** due to fluctuations and spatial correlations

Distribution P(Z/Z_0) acquires a **low-Z tail**:

$$P\,(Z) \propto Z^{\alpha-1}$$



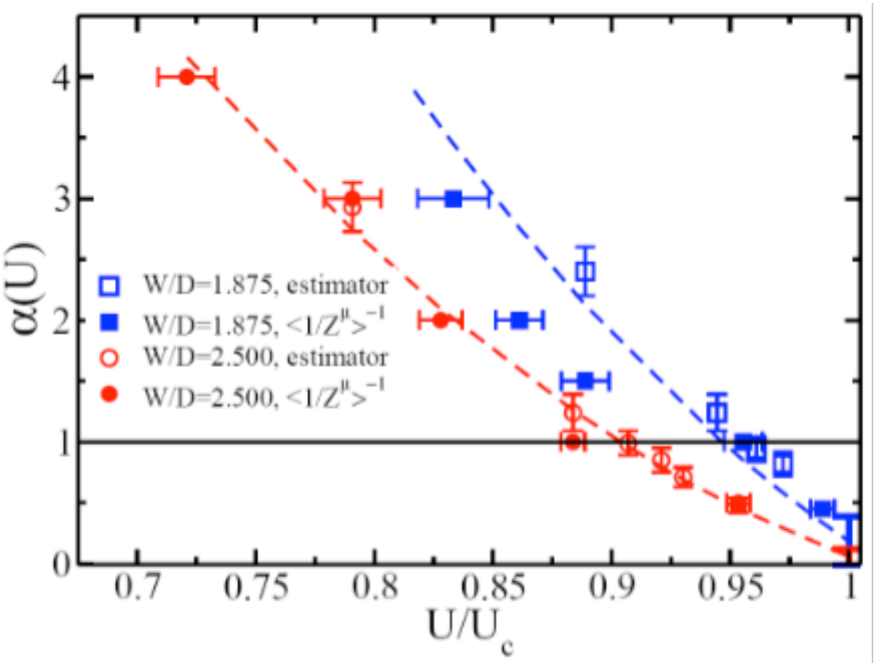
Results: Thermodynamics

- Remembering that the local Kondo temperature and $T_{Ki} \propto Z_i$

$$\chi_i(T) \sim \frac{1}{T + T_{Ki}} \Rightarrow \langle \chi(T) \rangle \sim \int dT_k \frac{T_K^{\alpha-1}}{T + T_K} \sim T^{\alpha-1}$$

Singular thermodynamic response

The exponent α is a function of disorder and interaction strength. **α=1** marks the onset of singular thermodynamics.

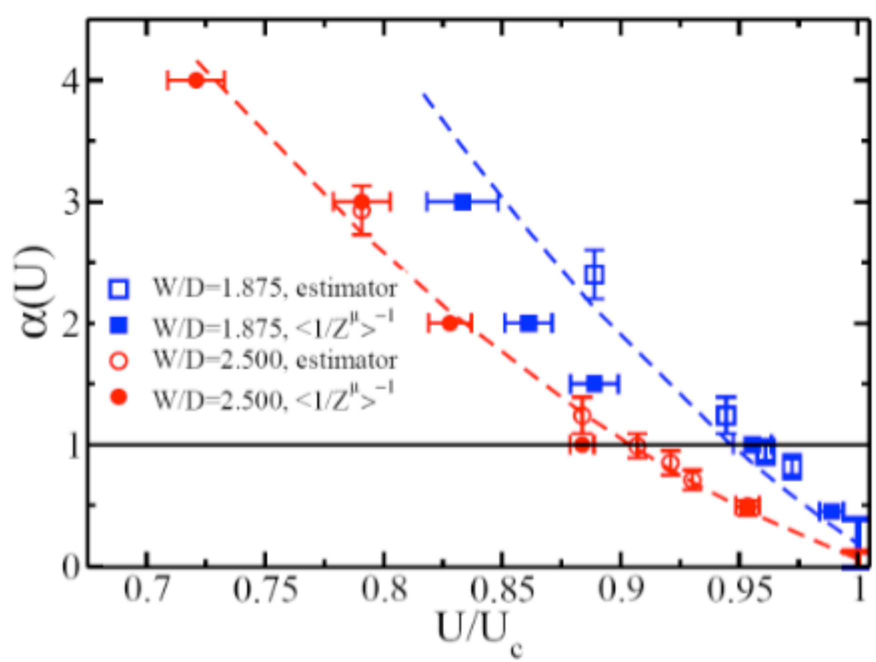


Quantum Griffiths phase

E. Miranda and V. D., Rep. Progr. Phys. **68**, 2337 (2005); T. Vojta, J. Phys. A **39**, R143 (2006)

Infinite randomness at the MIT?

- Most characterized Quantum Griffiths phases are precursors of a critical point where the effective disorder is infinite (D. S. Fisher, PRL **69**, 534 (1992); PRB **51**, 6411 (1995); ….)



$$P(Z) \propto Z^{\alpha-1}$$

$$\alpha \to 0 \Rightarrow \Delta Z \to \infty$$

Compatible with infinite randomness fixed point scenario

$1/\alpha$ – variance of log(Z)

“Size” of the rare events?

$$\boxed{\chi_i \sim Z_i^{-1}}$$

Replace the environment of given site outside square by uniform (DMFT-CPA) effective medium.

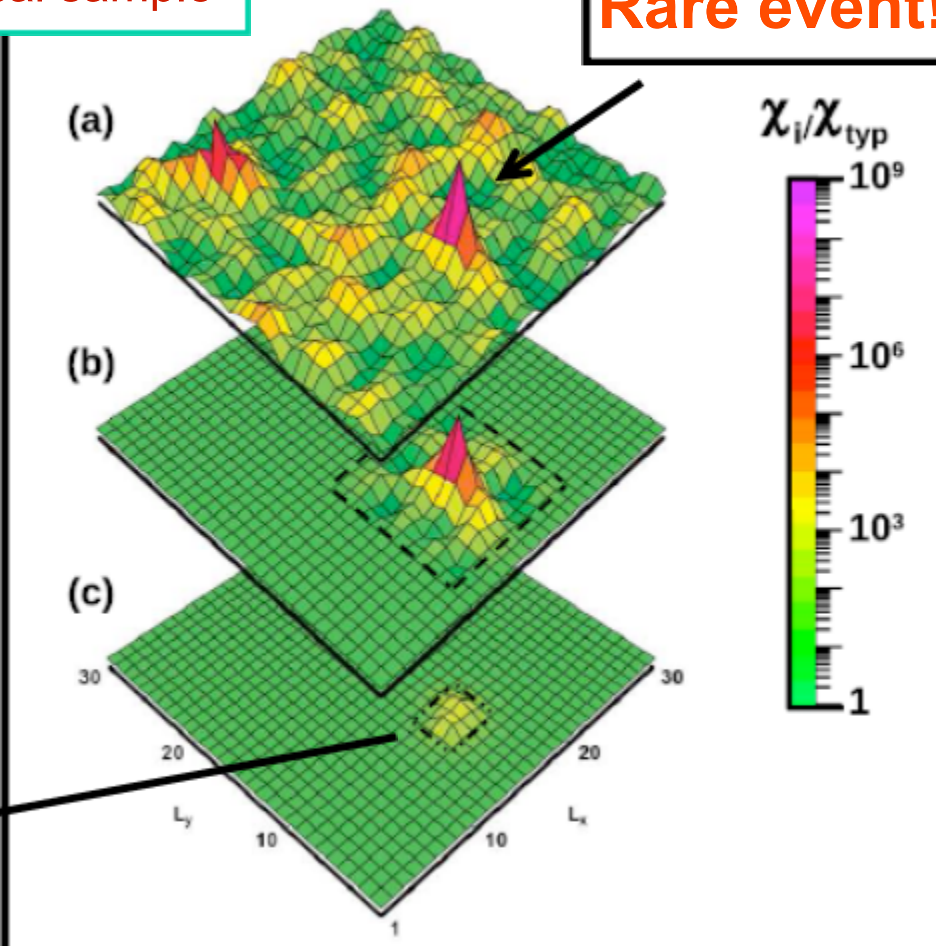
Reduce square size down to DMFT limit.

Rare events due to rare regions with weaker disorder

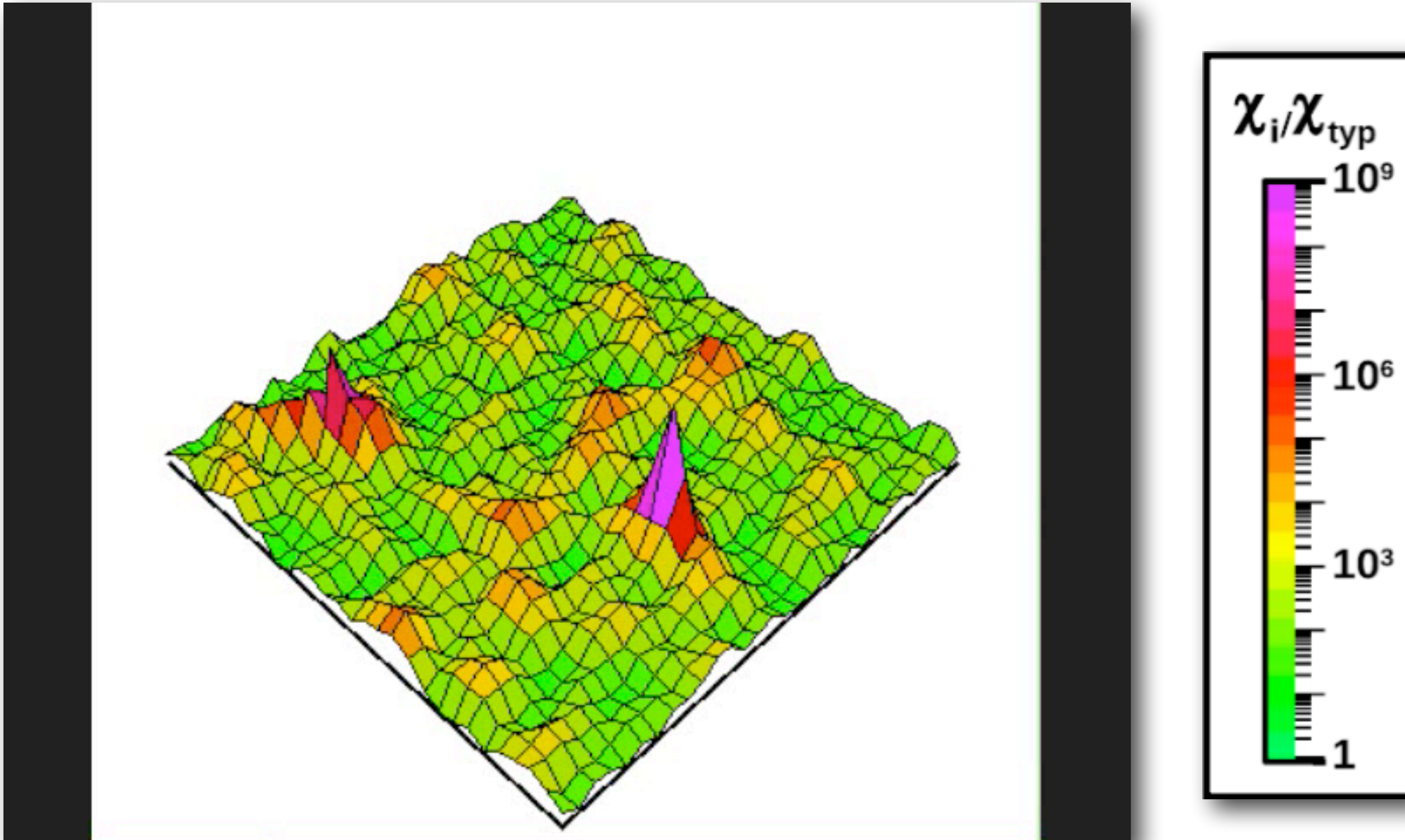
The rare event is preserved for a box of size $l > 9$: **rather smooth profile with a characteristic size.**

Typical sample

Rare event!



“Size” of the rare events: a movie



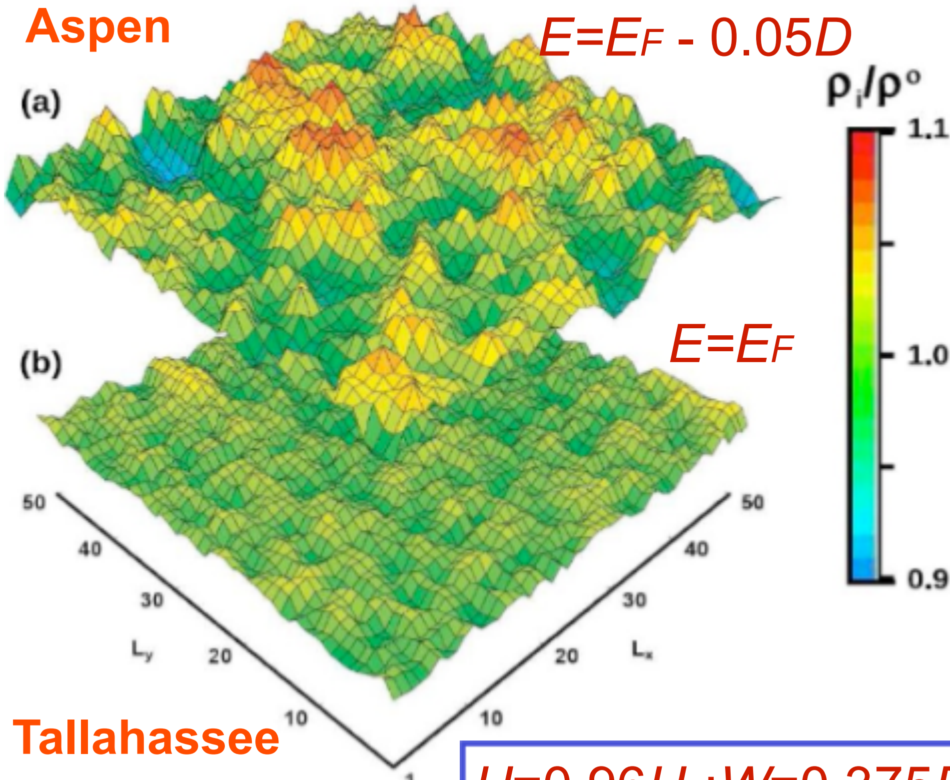
Killing the Mott droplet

Energy-resolved inhomogeneity!

- However, the effect is lost even slightly away from the Fermi energy:

Aspen

$E=E_F$ - 0.05D



$E=E_F$

Tallahassee

U=0.96U_c;W=0.375D

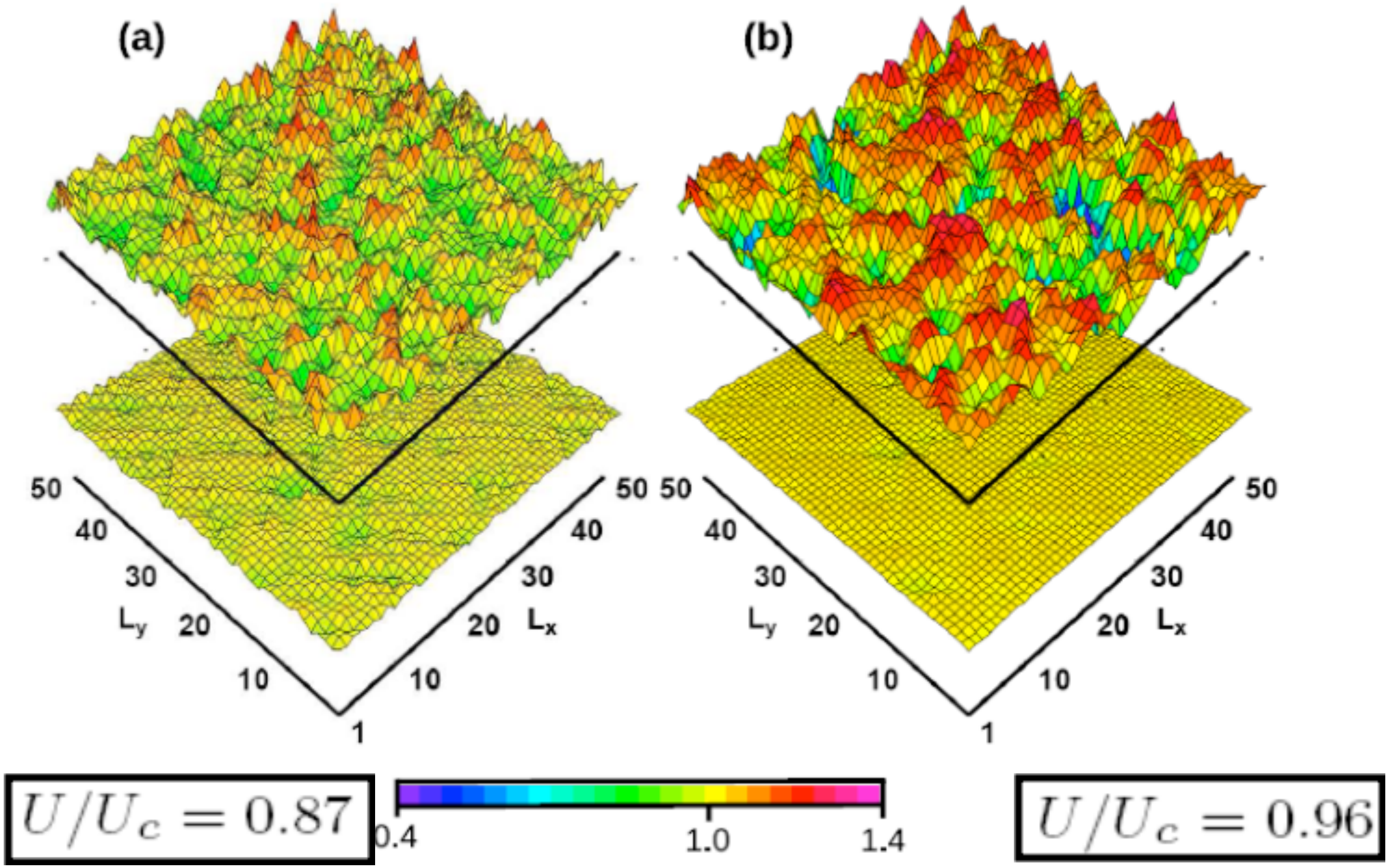
$$
\begin{aligned}
v_i(\omega \neq 0) &= \varepsilon_i + \Sigma_i(\omega \neq 0) \\
&= v_i + \omega\left(1 - Z_i^{-1}\right)
\end{aligned}
$$

The strong disorder effects reflect the wide fluctuations of Z_i

Similar to high-Tc materials, as seen by STM
Experiment: Seamus Davis (2005)
Theory: Garg, Trivedi, Randeria (2008))

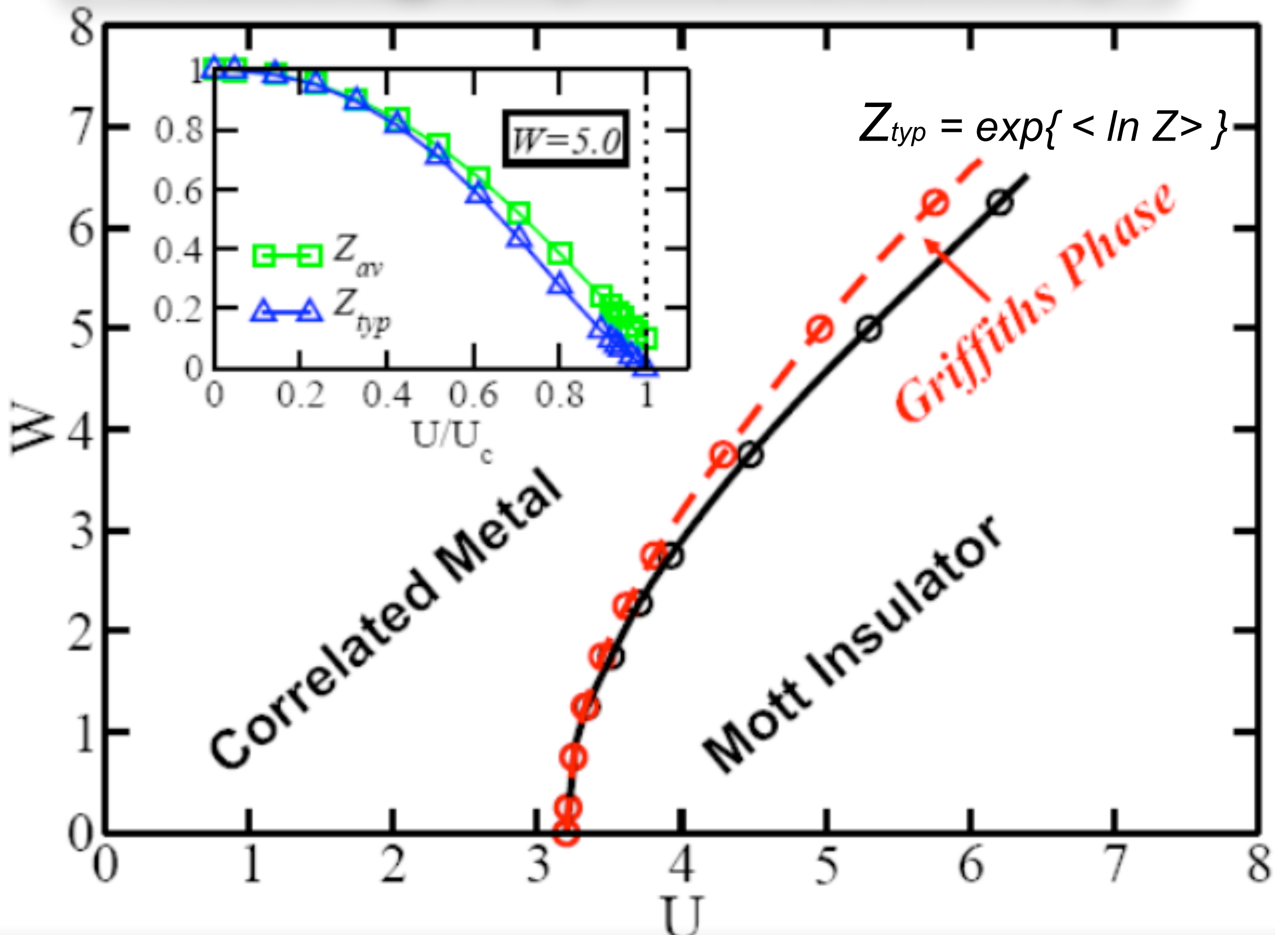
Generic to the strongly correlated materials?

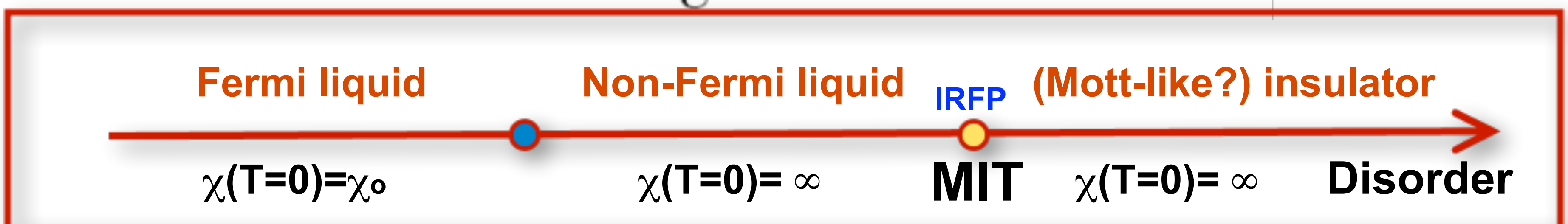
Mottness-induced contrast



Generic feature of all Mott systems, not only high Tc cuprates?!

Phase Diagram (moderate disorder)





NFL in Heavy Fermion Quasi-Crystals?

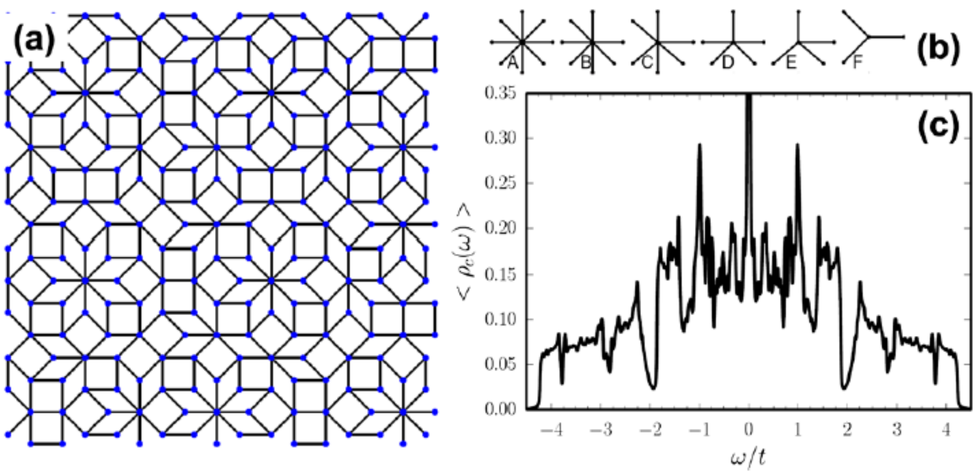
PRL 115, 036403 (2015) PHYSICAL REVIEW LETTERS week ending 17 JULY 2015

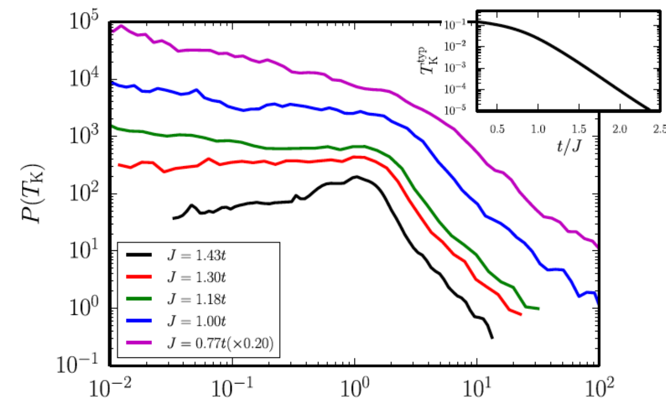
Non-Fermi-Liquid Behavior in Metallic Quasicrystals with Local Magnetic Moments

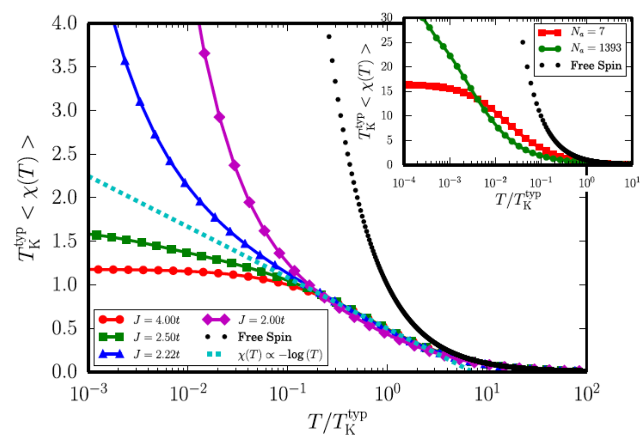
Eric C. Andrade,[1] Anuradha Jagannathan,[2] Eduardo Miranda,[3] Matthias Vojta,[4] and Vladimir Dobrosavljević[5]

New Materials: Rare-earth inter-metallics quasi-crystals

$Au_{51}Al_{34}Yb_{15}$

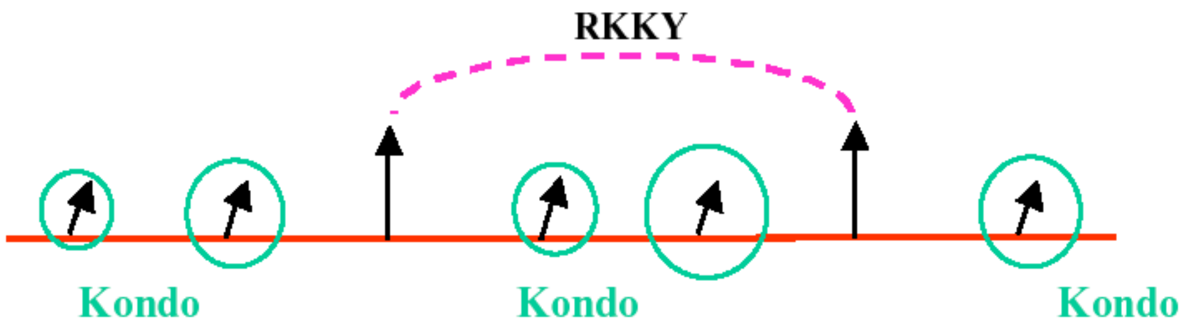






Aperiodic system - distribution of Kondo temps. T_K
Electronic Griffiths Phase, similar to disordered HF

Adding RKKY interaction: spin glass instability



- **RKKY interactions between (distant) low-T_K (unscreened) spins: oscillatory with distance ⇒ random in magnitude and sign**
- **Expect quantum spin-glass (SG) dynamics at low T**

EDMFT theory for RKKY interactions:

Bosonic bath: $$S_{RKKY} = g \int d\tau d\tau' \vec{\sigma}_f(\tau) \chi(\tau - \tau') \vec{\sigma}_f(\tau')$$

Self-consistency: $$\chi(\tau - \tau') = \overline{\langle \vec{\sigma}_f(\tau) \vec{\sigma}_f(\tau') \rangle}$$

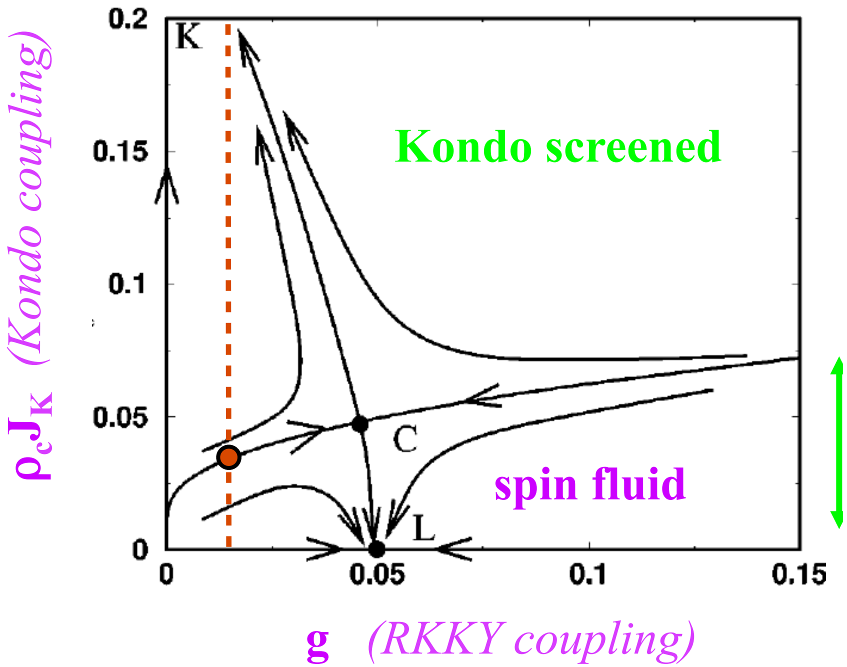
<u>Bose-Fermi (BF) Kondo model:</u> additional dissipation from bosonic bath!

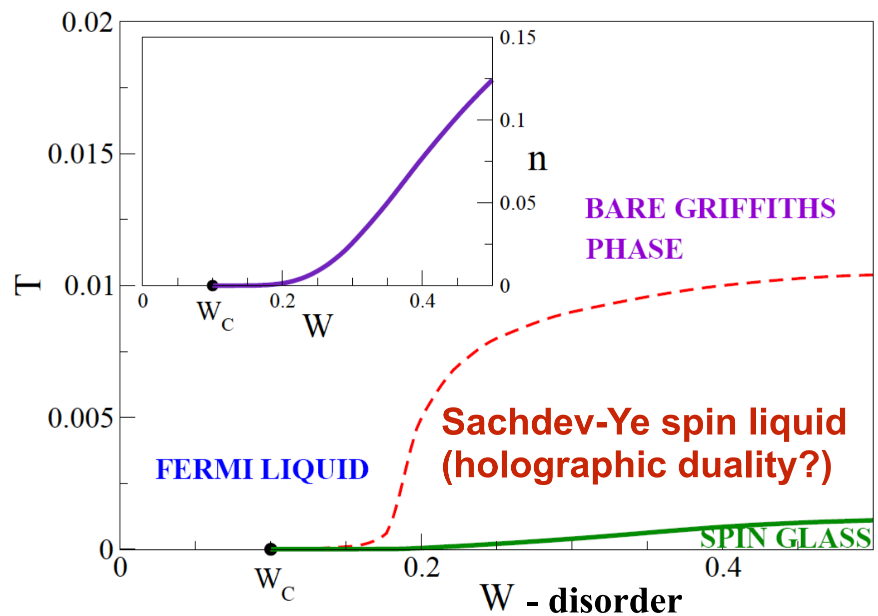
Fractionalization and Two-Fluid Behavior

PRL 95, 167204 (2005) PHYSICAL REVIEW LETTERS week ending 14 OCTOBER 2005

Spin-Liquid Behavior in Electronic Griffiths Phases

D. Tanasković,[1] V. Dobrosavljević,[1] and E. Miranda[2]





Analytical insight at weak disorder

PRL 104, 236401 (2010) PHYSICAL REVIEW LETTERS week ending 11 JUNE 2010

Quantum Ripples in Strongly Correlated Metals

E. C. Andrade,[1,2] E. Miranda,[1] and V. Dobrosavljević[2]

One impurity – **Friedel oscillations**

Interference: quantum corrections

Ballistic: $\Delta\sigma \sim T$ (d=2)
Diffusive: $\Delta\sigma \sim \log T$ (d=2)

(Aleiner, 2001)

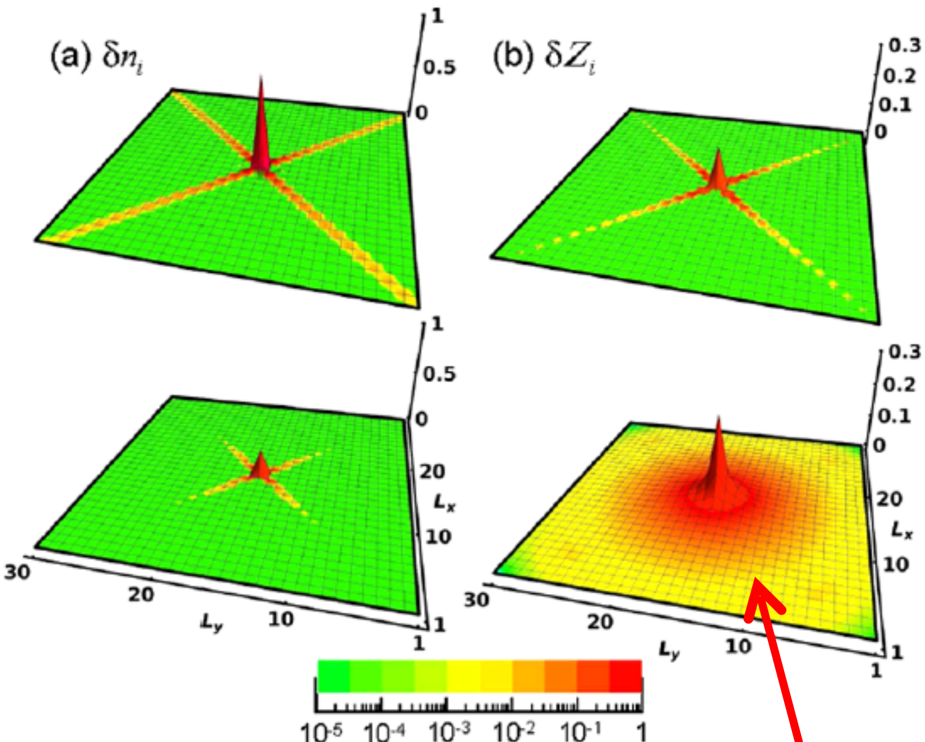
Weak impurity - analytic (perturbative) solution, numerics - general

Reduce to standard Hartree-Fock results at small U

Correlated regime and nonlocal terms??

Spatially correlated density fluctuations and Gutzwiller factors Z_i

"Healing" length and Mott quantum criticality



"healing" length

$$\delta Z_i \sim \frac{1-u}{U_c^2}\left(\frac{\pi^{(1-d)/2}}{2^{(1+d)/2}\xi^{(d-3)/2}}e^{-r_{ij}/\xi} - 4\,(1-u)^3\left[\mathbf{\Pi}^{(0)}\right]^{-1}_{ij}\right)\varepsilon_j^2$$

"healing"

Lindard function

$$\xi = \left(2z(1-u)\right)^{-1/2}$$

"healing" length

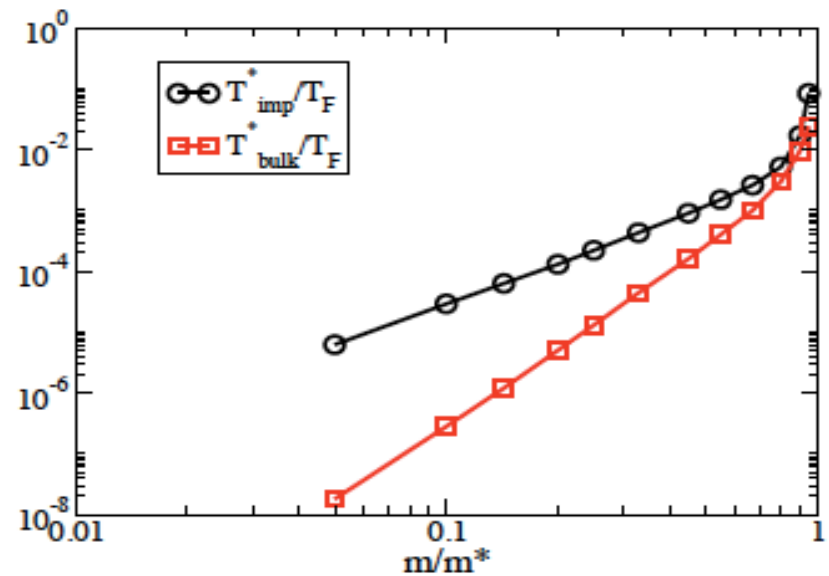
Quantum corrections vs. inelastic scattering

2D ballistic regime

$$\tau_{\mathrm{tr}}^{-1}(T) = \tau_0^{-1} A^2(u) \left\{ 1 + 2\frac{T}{T_F}\alpha(u)\, w(T, \gamma(T)) \right\} + \eta\gamma(T),$$

$$w(T,\gamma) = \int \frac{dx}{4} \mathrm{Sech}^2\left[\frac{x}{2}\right] \mathrm{Re}\left[\ln\Gamma\left(\frac{1}{2} + \frac{\gamma(T)}{2\pi T} + i\frac{x}{2\pi}\right)\right] + \frac{1}{2}\ln(2\pi) + \frac{\gamma(T)}{2\pi T}\ln\left(\frac{T_F}{2\pi T}\right)$$

Linear T transport only at $T < T^* \ll T_F$



Inelastic (electron-electron) scattering

$$\gamma(T) = C\,\Lambda(u)\, T_F\, (T/T_F)^2$$

$$\boxed{\Lambda(u) \sim (1-u)^{-2}}$$

PHYSICAL REVIEW B **91**, 020501(R) (2015)

Mottness-induced healing in strongly correlated superconductors

Shao Tang,[1] E. Miranda,[2] and V. Dobrosavljevic[1]

$$H = -\sum_{ij\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + J \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + \sum_i (\epsilon_i - \mu_0) n_i$$

Doped Mott insulator + weak disorder

D-wave pairing, large-N; Kotliar-Liu, 1992

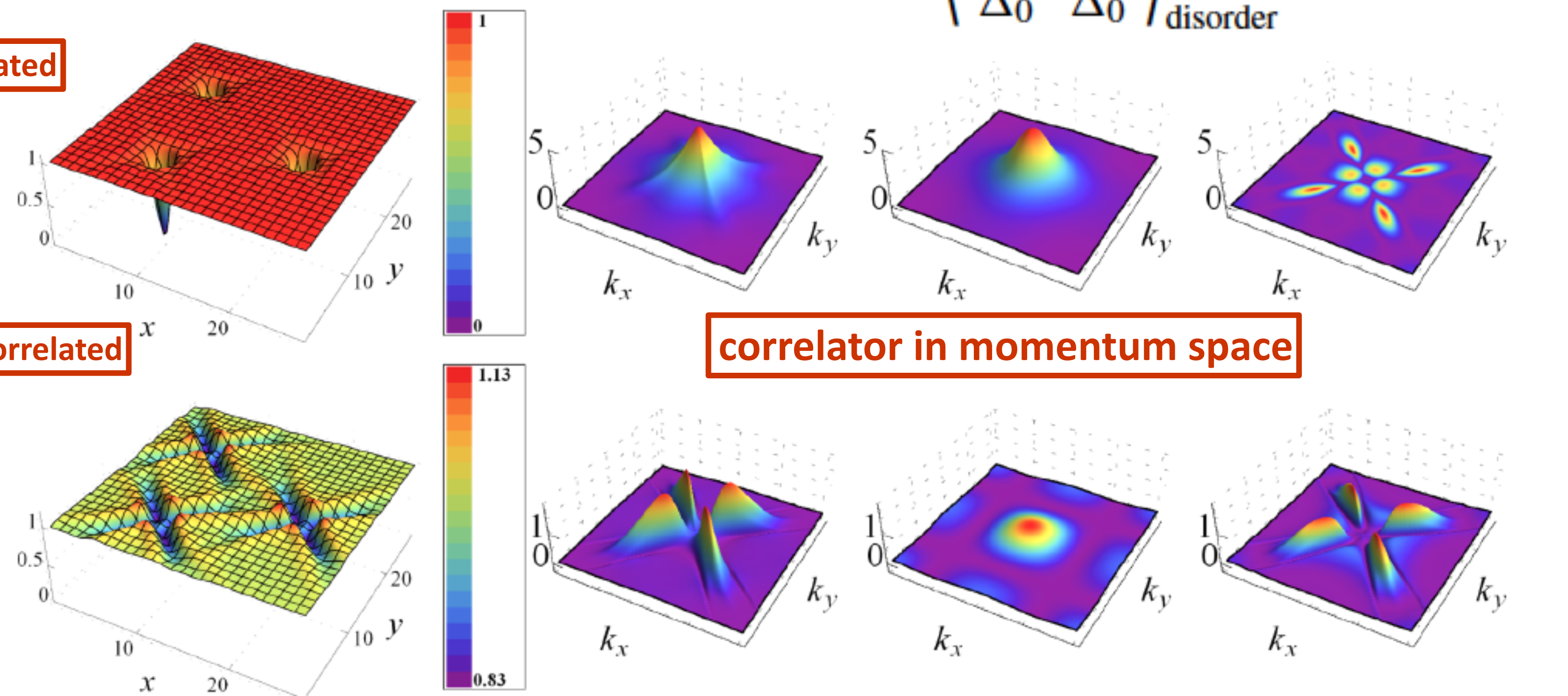
gap function

$$\Delta_{ij} = \langle f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow} \rangle \qquad \left\langle \frac{\delta \Delta_i}{\Delta_0} \frac{\delta \Delta_j}{\Delta_0} \right\rangle_{\text{disorder}} = f\left(\mathbf{r}_i - \mathbf{r}_j\right)$$

correlated

un-correlated

correlator in momentum space



"Healing" vs. Abrikosov Gor'kov pair-breaking?

PHYSICAL REVIEW B **93**, 195109 (2016)

Strong correlations generically protect d-wave superconductivity against disorder

Shao Tang,[1] V. Dobrosavljević,[1] and E. Miranda[2]

$$H = -\sum_{ij\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + J \sum_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + \sum_i (\epsilon_i - \mu_0) n_i$$

D-wave pairing, large-N; Kotliar-Liu, 1992

Doped Mott insulator + weak disorder

BCS+AG: Even non-magnetic impurities -> strong pair-breaking for D-wave??

T-matrix at weak disorder + strong correlations

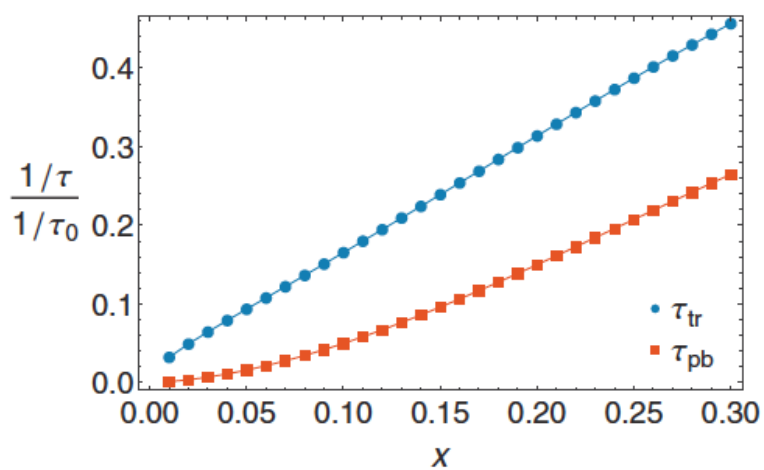
$$\ln \frac{T_{c0}}{T_c} = \psi\left(\frac{1}{2} + \frac{\alpha}{2}\right) - \psi\left(\frac{1}{2}\right)$$

$$\alpha \equiv \; 1/(2\pi T_c \tau_{pb})$$

$$\frac{1}{\tau_{pb}} = \frac{x^2 n m^*}{2\pi} \int_0^{2\pi} d\theta \, g\left[\left|\sin\left(\frac{\theta}{2}\right)\right|\right](1 - \cos 2\theta)$$

$$g(y) \equiv \frac{t^2}{\{\rho^* \lambda_0 k_F^2 y^2 g_L(y) + x[1 - 2\rho^* E_F g_L(y)]\}^2}$$

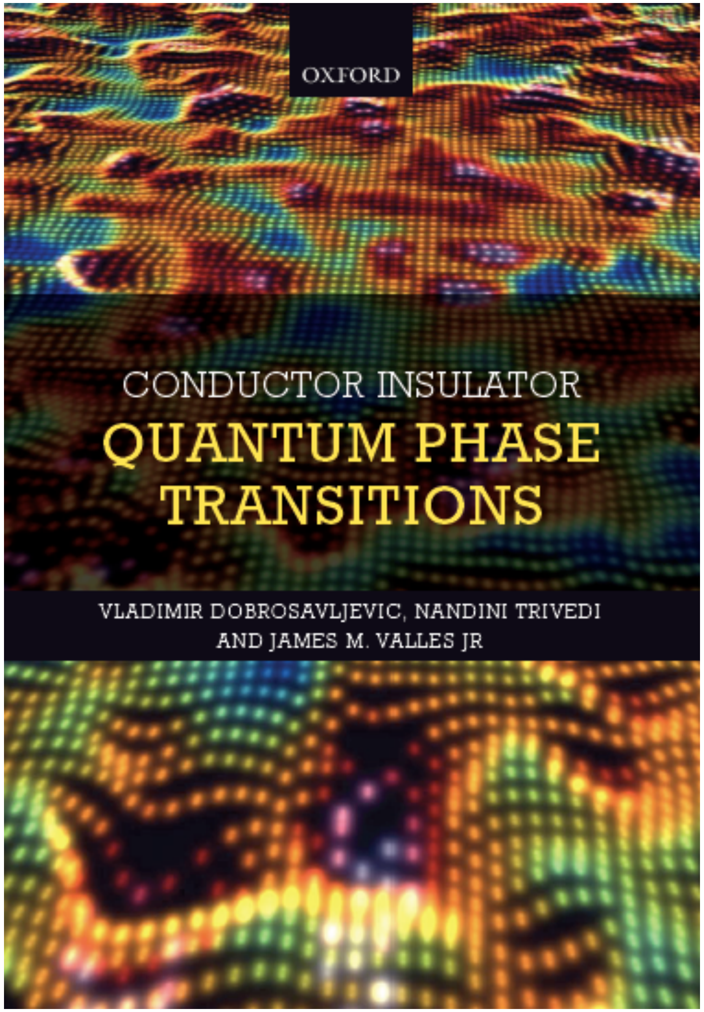
Strong suppression of pair-breaking!



Perspectives and challenges

- Significant disorder renormalization due to correlations
- Diffusion modes, “interaction-localization”?
- Disorder-induced non-Fermi liquid behavior
- Other non-periodic systems with correlations (HF quasi-crystals)?
- IRFP behavior, SDRG approaches?
- Inter-site (magnetic) correlations, CDMFT?
- Bosonic modes in “weak” FL, fractionalization, spin-glass EDMFT?
- Behavior out of equilibrium (MBL) with strong correlations and disorder?

To learn more:



http://badmetals.magnet.fsu.edu

(just Google "Bad Metals")

Book:

Oxford University Press, June 2012

Already listed on Amazon.com

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